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Compressible Laminar Boundary Layer Flow over a Yawed Infinite Cylinder with Heat Transfer via Quasilinearization Technique

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1. INTRODUCTION

It is well known that the systems of non-linear boundary value problems in their complete generality hardly lend themselves to analytical approaches. Consequently, one resorts to some numerical technique that helps to compute their solutions at a discrete set of nodal points. In the last two decades many numerical methods, as surveyed and classified by Aktas and Stetter [1] and later reviewed by Fox [6], with varying degrees of success have been developed for solving these non-linear boundary-value problems. But while employing these methods to systems of engineering interest, one usually confronts difficulties of various types, such as, for example, specification of boundary conditions at infinity, changes in the order of differential system depending upon the values of some parameter involved, and the coupled nature and high order of the system that renders it unmanageable to handle them analytically and numerically. These difficulties aside, the non-linearity of the differential system makes it imperative to use some kind of iterative process which can be initiated only when the initial approximations of the unknown solutions are available. Moreover, in view of the boundary-value nature of the system one is compelled to guess the missing values of the field variables at the initial point that often leads to the ill-conditioning phenomenon of the resulting algebraic system[1].

The method of quasilinearization partially overcomes the above difficulties (Bhutani *et al.* [3]). As a sequel to that work on the utilization of quasilinearization technique for solving non-linear two-point boundary-value problems we have herein endeavoured to test the efficacy and

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[1] This may happen as and when these guesses are not close enough to the missing actual values.

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reliability of this technique on a flow problem from the realm of three-dimensional compressible boundary layer—an area that encompasses all the complications of the compressible boundary layer and that due to the three-dimensional nature of the flow. More specifically, we have here carried over the said technique to investigate the compressible laminar boundary layer flow over a yawed infinite cylinder with heat transfer and arbitrary Prandtl number. Owing to its technological importance and the large stakes involved, the problem received due attention at the hands of NACA experts, who, to ensure the accuracy and reliability of the numerical solutions, solved it independently by two different techniques at two different laboratories of NACA. Though the two approaches adopted by them [17] yielded practically the same results, yet, in essence, both of them boiled down to the trial-and-error methods. This prompted us to re-examine the problem through quasilinearization technique, the efficacy of which for solving highly complicated non-linear boundary-value problems has already been demonstrated by Huang [8–10], Lee *et al.* [13], and Bhutani *et al.* [3]. However, all these works invariably required a priori knowledge, however crude, of the unknown initial profiles. The remarkable feature of the present study is that besides establishing the efficacy and reliability of the quasilinearization technique we have herein dispensed with the requirement of a priori knowledge of the initial profiles.

2. Problem Formulation and Similarity Equations

The differential system governing the three-dimensional compressible laminar boundary layer flow over a yawed infinite cylinder with heat transfer and arbitrary Prandtl number, in its complete generality, has been discussed by many authors [15, 17, 21] and, therefore, is not reproduced here. The physical assumptions and notations used in this analysis are those of Reshotko and Beckwith [17]. Assuming the linear viscosity–temperature relationship, introducing a stream function ψ, and using the modified Stewartson transformation, the boundary layer equations are first simplified through the transformed coordinates. On further assuming the external chordwise velocity distribution, U_e, to be of Falkner–Skan type

$$U_e = CX^m \tag{1}$$

along with the similarity transformations defined by

$$\begin{aligned} \psi &= f(\eta)\left(\frac{2\nu_0 U_e X}{m+1}\right)^{1/2}, \\ \eta &= Z\left(\frac{m+1}{2}\frac{U_e}{\nu_0 X}\right)^{1/2}, \end{aligned} \tag{2}$$

the resulting partial differential equations are reduced to a set of two-point non-linear boundary-value problem given by

$$f''' + ff'' = \beta\left[f'^2 - 1 - \left(\frac{T_0}{T_{\mathrm{N0}}} - 1\right)(1 - g^2) - \frac{T_0}{T_{\mathrm{N0}}}\left(\frac{T_{\mathrm{w}}}{T_0} - 1\right)(1 - \theta)\right], \tag{3}$$

$$g'' + fg' = 0, \tag{4}$$

$$\theta'' + \mathrm{Pr}\, f\theta' = \frac{1 - \mathrm{Pr}}{1 - T_{\mathrm{w}}/T_0}\left[\frac{\gamma - 1}{2}\left(\frac{u_{\mathrm{e}}}{a_0}\right)^2 (f'^2)'' + \left(1 - \frac{T_{\mathrm{N0}}}{T_0}\right)(g^2)''\right] \tag{5}$$

where

$$\beta = \frac{2m}{1 + m}. \tag{6}$$

The associated boundary conditions are transformed to

$$f = f' = g = \theta = 0 \quad \text{at} \quad \eta = 0 \tag{7}$$

$$f' = g = \theta = 1 \quad \text{as} \quad \eta \to \infty. \tag{8}$$

In the above equations the subscripts w, e, and 0 are used to indicate, respectively, the wall or surface value, the external or local flow at the outer edge of the boundary layer, and the free stream stagnation values. X and Z are the transformed chordwise and normal coordinates, g is the spanwise velocity variable, θ is the normalized enthalpy function, and both g and θ are functions of η; C is the stagnation line chordwise velocity gradient, a, T, and u are, respectively, the sonic velocity, static temperature, and chordwise velocity component where γ is the ratio of two specific heats and ν is the kinematic viscosity. It is interesting to note that

$$\frac{T_0}{T_{\mathrm{N0}}} = \frac{1 + ((\gamma - 1)/2)\, M_\infty^2}{1 + ((\gamma - 1)/2)\, M_\infty^2 \cos^2 \Lambda} \tag{9}$$

combines the effect of both yaw angle Λ and Mach number M in a single parameter.

The system of Eqs. (3)–(5) along with Eqs. (7)–(8) encompasses many important studies carried out from time to time. In particular, for the case of incompressible flow over a yawed infinite cylinder where $T_0 = T_{\mathrm{N0}} = T_{\mathrm{w}}$, Eq. (3) delinks from the rest of the system, thereby confirming that the chordwise flow both inside and outside the boundary layer is independent of the spanwise flow. In this case, one has to deal with a truncated uncoupled system of two boundary-value problems and each of them can be dealt with independently. This "independence principle" was observed almost simultaneously by Jones [11], Prandtl [16], Sears [20], and

Strumminski [22]. Further, through the physical considerations it can be seen that the cases $\beta < 0$, $\beta = 0$, $\beta = 1$, and $\beta \to 2$ (when $m \to \infty$) correspond, respectively, to an unfavourable pressure gradient, flat plate flow, stagnation line flow, and an infinitely favourable pressure gradient. The same independence principle holds for flat plate flow but does not hold for compressible flow with pressure gradient even for zero heat transfer and is discussed by Crabtree [5] and Tinkler [23].

It is worth noting that the two sides of Eq. (5) are not functionally consistent since u_e, in general, can be a function of x. In order to make the two sides consistent the right-hand side should be either zero or a function of η alone and to achieve this the following possibilities arise: (i) $u_e = 0$, (ii) u_e is a non-zero constant, (iii) $\mathrm{Pr} = 1$, (iv) $((\gamma - 1)/2)(u_e/a_0)^2 \to \cos^2 \Lambda$, and (iv) $\gamma = 1$. Keeping in view the physical realizability of the various parameters involved, the attention is confined to the situation where either $\mathrm{Pr} = 1$ or $u_e = 0$. For $\mathrm{Pr} = 1$ Eqs. (4) and (5) are identical, so that the distribution of the normalized enthalpy function θ is the same as the spanwise velocity function g, and one is left with a reduced system of only two equations. Its solution, for zero yaw, is extensively discussed by Li and Nagamatsu [14] and Cohen and Reshotko [4], whereas the case of heat-insulated wall characterized by $T_w = T_0$ is studied by Crabtree [5]. Furthermore, for zero yaw and insulated wall we are left with only one single equation, which is the same as that for incompressible flow past a wedge and has been studied in depth by Hartree [7] for different values of the pressure gradient β. On the other hand, consideration of zero u_e allows arbitrary Prandtl number and is studied by Reshotko and Beckwith [17] for stagnation line flow characterized by $m = \beta = 1$. Other cases of importance requiring consideration are those of arbitrary Prandtl number Pr and arbitrary pressure gradient characterized by β and are a subject matter of other study by us. Moreover, the consideration of constant u_e needs further investigation.

3. Method of Solution

In order to solve the non-linear boundary-value problem given by Eqs. (3)–(8) via quasilinearization technique we need to reduce Eqs. (3)–(5) to a system of first-order equations. To this end we introduce the transformation

$$(f, f', f'', g, g', \theta, \theta') = (x_1, x_2, x_3, x_4, x_5, x_6, x_7) = X^{\mathrm{T}}, \tag{10}$$

which immediately leads us to arrive at the following system of first-order differential equations,

$$
\begin{aligned}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= x_3, \\
\dot{x}_3 &= -x_1 x_3 + B_0 x_2^2 + B_1 x_4^2 + B_2 x_6 - B_8, \\
\dot{x}_4 &= x_5, \\
\dot{x}_5 &= -x_1 x_5, \\
\dot{x}_6 &= x_7, \\
\dot{x}_7 &= -\Pr x_1 x_7 + B_3 x_3^2 - B_3 x_1 x_2 x_3 + B_4 x_2^3 + B_5 x_2 x_4^2 \\
&\quad + B_6 x_2 x_6 + B_7 x_5^2 - B_7 x_1 x_4 x_5 - B_9 x_2
\end{aligned}
\tag{11}
$$

where the dot stands for differentiation with respect to η and the constants Bs, expressible in terms of the basic parameters, β, T_0/T_{N0}, T_{w}/T_0, Pr, γ, u_{e}, and a_0, are given by

$$
\begin{aligned}
B_0 &= \beta, \\
B_1 &= \left(\frac{T_0}{T_{\mathrm{N0}}} - 1\right)\beta, \\
B_2 &= \frac{T_0}{T_{\mathrm{N0}}}\left(\frac{T_{\mathrm{w}}}{T_0} - 1\right)\beta, \\
B_3 &= 2\,\frac{1-\Pr}{1 - T_{\mathrm{w}}/T_0}\left(\frac{\gamma-1}{2}\right)\left(\frac{u_{\mathrm{e}}}{a_0}\right)^2, \\
B_4 &= B_3\beta, \\
B_5 &= \left(\frac{T_0}{T_{\mathrm{N0}}} - 1\right)B_4, \\
B_6 &= \frac{T_0}{T_{\mathrm{N0}}}\left(\frac{T_{\mathrm{w}}}{T_0} - 1\right)B_4, \\
B_7 &= 2\left(1 - \frac{T_{\mathrm{N0}}}{T_0}\right)\frac{1-\Pr}{1 - T_{\mathrm{w}}/T_0}, \\
B_8 &= B_0 + B_1 + B_2,
\end{aligned}
$$

and

$$B_9 = B_4 + B_5 + B_6.$$

If n is the iteration number, then following Bellman and Kalaba [2] and Lee [12], the quasilinearized version of Eq. (11) can be expressed as the matrix equation

$$\dot{X}^{n+1} = AX^{n+1} + B, \qquad n = 0, 1, 2, \ldots. \tag{12}$$

In Eq. (12) the non-zero elements of the square matrix A and the column matrix B are given by

$$a_{12} = a_{23} = a_{45} = a_{67} = 1,$$

$$a_{33} = a_{55} = -x_1^n,$$

$$a_{31} = -x_3^n,$$

$$a_{32} = 2B_0 x_2^n,$$

$$a_{34} = 2B_1 x_4^n,$$

$$a_{36} = B_2,$$

$$a_{51} = -x_5^n,$$

$$a_{71} = -\Pr x_7^n - B_3 x_2^n x_3^n - B_7 x_4^n x_5^n,$$

$$a_{72} = -B_3 x_1^n x_3^n + 3B_4 (x_2^n)^2 + B_5 (x_4^n)^2 + B_6 x_6^n - B_9,$$

$$a_{73} = 2B_3 x_3^n - B_3 x_1^n x_2^n,$$

$$a_{74} = 2B_5 x_2^n x_4^n - B_7 x_1^n x_5^n,$$

$$a_{75} = 2B_7 x_5^n - B_7 x_1^n x_4^n,$$

$$a_{76} = B_6 x_2^n,$$

$$a_{77} = -\Pr x_1^n,$$

$$b_3 = -B_0 (x_2^n)^2 + x_1^n x_3^n - B_1 (x_4^n)^2 - B_8,$$

$$b_5 = x_1^n x_5^n,$$

and

$$\begin{aligned} b_7 = {} & 2B_3 x_1^n x_2^n x_3^n - 2B_4 (x_2^n)^3 - B_3 (x_3^n)^2 - 2B_5 x_2^n (x_4^n)^2 \\ & + 2B_7 x_1^n x_4^n x_5^n - B_7 (x_5^n)^2 - B_6 x_2^n x_6^n + \Pr x_1^n x_7^n. \end{aligned}$$

In view of the transformation (10) the boundary conditions (7)–(8) translate, rspectively, to

$$x_1^n = x_2^n = x_4^n = x_6^n = 0 \qquad \text{at} \quad \eta = 0 \tag{13}$$

$$x_2^n = x_4^n = x_6^n = 1 \qquad \text{as} \quad \eta \to \infty. \tag{14}$$

To obtain the solution of Eqs. (12) subject to the initial conditions (13) a particular solution $P^{n+1}(\eta)$ of Eqs. (12) is generated with the initial conditions

$$P^{n+1}(0) = (0, 0, 0, 0, 0, 0, 0)^{\mathrm{T}}. \tag{15}$$

Further, corresponding to the three missing initial conditions, namely, x_3, x_5, x_7, three homogeneous solutions $H_3^{n+1}(\eta)$, $H_5^{n+1}(\eta)$, $H_7^{n+1}(\eta)$ of the auxiliary equation associated with Eq. (12) with $B=0$ are generated. The initial conditions to which these homogeneous solutions are subjected are given by

$$H_i^{n+1}(0) = (\delta_{ij})^{\mathrm{T}}, \qquad i=3, 5, 7; \quad j=1, 2, \ldots, 7 \tag{16}$$

where δ_{ij} is Kronecker's delta equal to 1 for $i=j$ and 0 otherwise. As in [3] all these solutions, particular and homogeneous, are obtained by the fourth-order Runge–Kutta method and then linearly combined to give the general solution of Eqs. (12) subject to conditions (13) in the form

$$X^{n+1}(\eta) = P^{n+1}(\eta) + c_2 H_3^{n+1}(\eta) + c_4 H_5^{n+1}(\eta) + c_6 H_7^{n+1}(\eta) \tag{17}$$

where c_2, c_4, and c_6 are the constants of integration to be determined in such a way that Eqs. (17) satisfy the boundary conditions (14). Thus, one is led to the linear algebraic system

$$\bar{H}\bar{c} = \bar{P} \tag{18}$$

where the square matrix $\bar{H}$ and $\bar{c}$, $\bar{P}$, the column vectors of order 3, obtainable from Eqs. (17) and (14), are given by

$$\bar{c} = \begin{bmatrix} c_2 \\ c_4 \\ c_6 \end{bmatrix}; \qquad \bar{P} = \begin{bmatrix} 1 - P_2^{n+1}(\eta) \\ 1 - P_4^{n+1}(\eta) \\ 1 - P_6^{n+1}(\eta) \end{bmatrix}_{\eta=\infty}$$

and

$$\bar{H} = \begin{bmatrix} H_{32}^{n+1} & H_{52}^{n+1} & H_{72}^{n+1} \\ H_{34}^{n+1} & H_{54}^{n+1} & H_{74}^{n+1} \\ H_{36}^{n+1} & H_{56}^{n+1} & H_{76}^{n+1} \end{bmatrix}_{\eta=\infty} \tag{19}$$

In Eqs. (19), $P_i^{n+1}(\eta)$, $H_{3i}^{n+1}(\eta)$, $H_{5i}^{n+1}(\eta)$, and $H_{7i}^{n+1}(\eta)$ are the ith components, respectively, of $P^{n+1}(\eta)$, $H_3^{n+1}(\eta)$, $H_5^{n+1}(\eta)$, and $H_7^{n+1}(\eta)$. For the applicability of quasilinearization technique the square matrix $\bar{H}$ must be non-singular and this also turns out to be the condition for the existence of the solution of the system (18). The solution of system (18) yields the constants c_2, c_4, c_6 which, in turn, determines the next approximation to the solution of Eqs. (3)–(5) subject to the conditions (7) and (8). The process is iterative in nature and provides the solutions of Eqs. (3)–(5) if one knows the initial profile X^0. Through our experience of working on the applicability of quasilinearization technique we have observed that if X^0, the initial approximation of the unknown solution, is so chosen that it satisfies the given boundary conditions and adheres to the internal consistency of the field variables, then the quasilinearization technique converges. In the case under consideration the initial approximations are assumed to be

$$\begin{aligned} x_1^0(\eta) &= \eta^2/2L, \\ x_2^0(\eta) &= \eta/L = x_4^0(\eta) = x_6^0(\eta), \\ x_3^0(\eta) &= 1/L = x_5^0(\eta) = x_7^0(\eta) \end{aligned} \tag{20}$$

where L is the parameter that controls the duration of integration and is to be chosen in such a way that the boundary conditions (14) are smoothly satisfied. For the present case it is found that $L = 6$ is a sufficiently large interval in which all the required boundary conditions are satisfied.

4. Discussion of Numerical Results

The first particular case of interest is that of incompressible wedge flow with arbitrary pressure gradient and is readily deducible from Eqs. (3)–(5) by taking $u_e = 0$, $\Pr = T_0/T_{N0} = T_w/T_0 = 1$ for arbitrary β. The computed results (Fig. 1) agree well with those of Hartree [7] as reported by Schlichting [19]. The other particular case that has drawn attention of many a workers in this area is that of compressible boundary layer flow with heat transfer and arbitrary pressure gradient across the wall kept at a constant temperature, S_w. This can be obtained from Eqs. (3)–(5) by putting $\Lambda = 0$, corresponding to zero yaw, $T_0/T_{N0} = 1$, $u_e = 0$, $\Pr = 1$ and using the following transformation for the enthalpy distribution:

$$\begin{aligned} S(\eta) &= \frac{T_w}{T_0} - \left(\frac{T_w}{T_0} - 1\right)\theta, \\ S_w &= \frac{T_w}{T_0}. \end{aligned} \tag{21}$$

Corresponding to different values of β and S_w, a large number of results have been computed and for comparison purposes the values of $f''(0)$, $S'(0)$ for various values of wall temperature, $0 \leqslant S_w \leqslant 2$, and $\beta = 0.5$ are tabulated in Table I. The comparison with Li and Nagamatsu [14] values as given by Stewartson [21] is quite up to the mark.

The graphs depicting the variations of f', f'', and S with respect to β $(-0.325 \leqslant \beta \leqslant 2)$ for all three different states of wall—cool $(S_w = 0.2)$, adiabatic $(S_w = 1)$, and hot $(S_w = 2.0)$—tally quite well with those of

TABLE I

$\mathrm{Pr} = 1, \quad \beta = 0.5, \quad T_0/T_{N0} = 1$

	$f''(0)$		$S'(0)$	
$S_w = \dfrac{T_w}{T_0} - 1$	Cohen and Reshotko	Modified (present study)	Cohen and Reshotko	Modified (present study)
0.0	0.5806	0.58174	0.4948	0.49466
0.2	0.6547	0.65525	0.403	0.40373
0.6	0.7947	0.79550	0.209	0.20921
1.0	0.9277	0.92802	0.0	0.00000
2.0	1.2351	1.23524	−0.5725	−0.57309

Cohen and Reshotko [4] and some of them are reproduced in Figs. 1 – 8. Their remarks concerning the velocity overshoot in the case of a heated wall with favourable pressure gradient (Fig. 6), the comparison of velocity profile with the enthalpy profile in the presence of external pressure gradient, and occurrence of maximum shear stress at the wall for favourable pressure gradient and its movement away from the wall for unfavourable pressure gradient, and the movement becoming pronounced with the magnitude of unfavourable pressure gradient, stand verified (see Fig. 2).

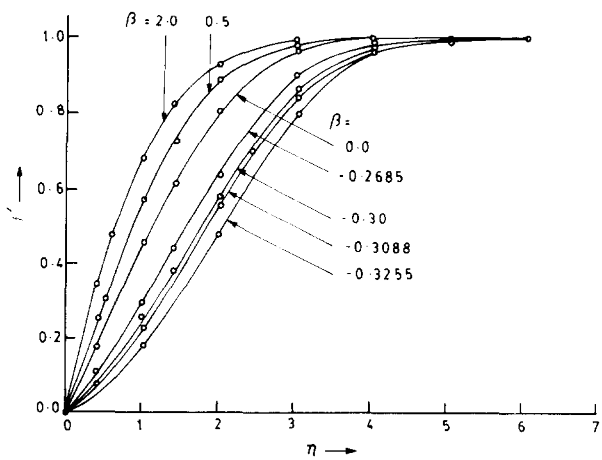

FIG. 1. Velocity distribution for a cool wall, $S_w = 0.2$.

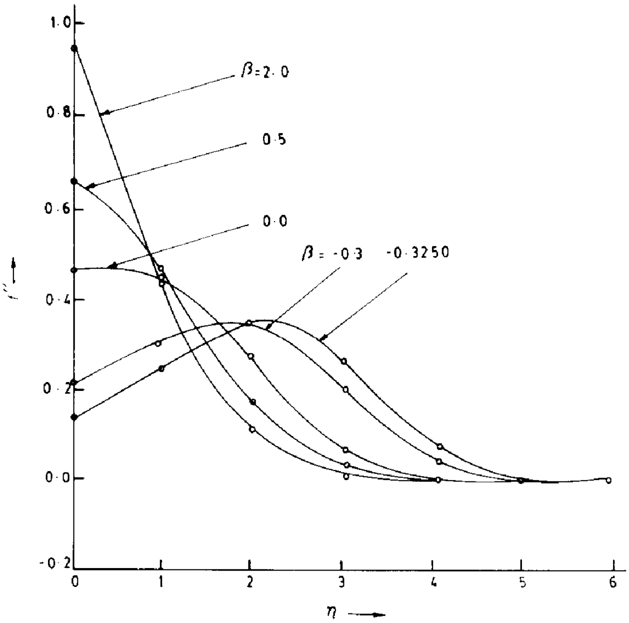

FIG. 2. Distribution of shearing stress for cool wall, $S_w = 0.2$.

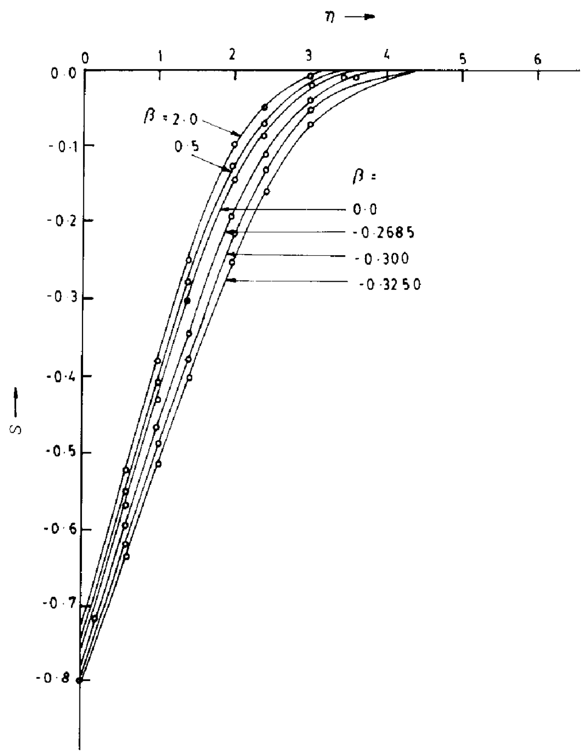

FIG. 3. Enthalpy distribution for a cool wall, $S_w = 0.2$.

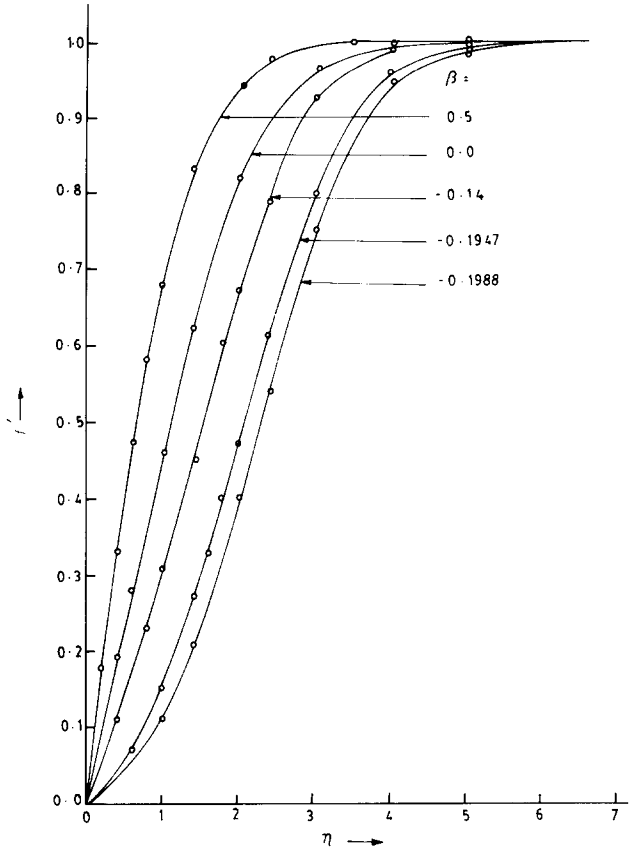

FIG. 4. Velocity distribution for an adiabatic wall, $S_w = 0$.

Moreover, it is observed that for an adiabatic as well as for a heated wall the convergence of the iterative process is significantly decelerated as the magnitude of the adverse pressure gradient increases. Further, in the case of the adiabatic wall a convergence of an order of -3 could be attained for $\beta = -0.1988$, the value where separation of flow takes place. On the other hand, for a cool wall convergence remains of high order, for all values of β.

Finally, due attention was paid to the computational aspect of the problem of yawed infinite wing in the compressible flow. All the quantities of aerodynamical interest are obtained for the stagnation line flow, characterized by $\beta = 1$. It is seen that the computed values, obtained through quasilinearization, agree with those of Reshotko and Beckwith [17] at least up to 2 decimal places. Table II presents the corresponding results for $\mathrm{Pr} = 0.7$ and $u_e = 0$.

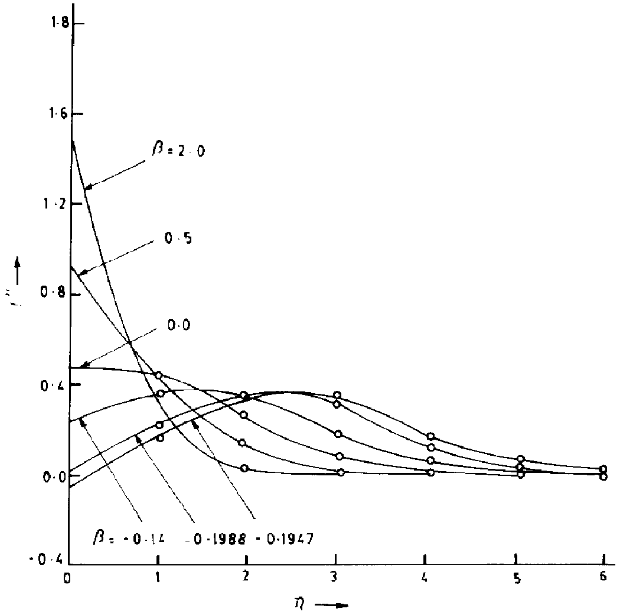

FIG. 5. Enthalpy distribution for an adiabatic wall, $S_w = 0$.

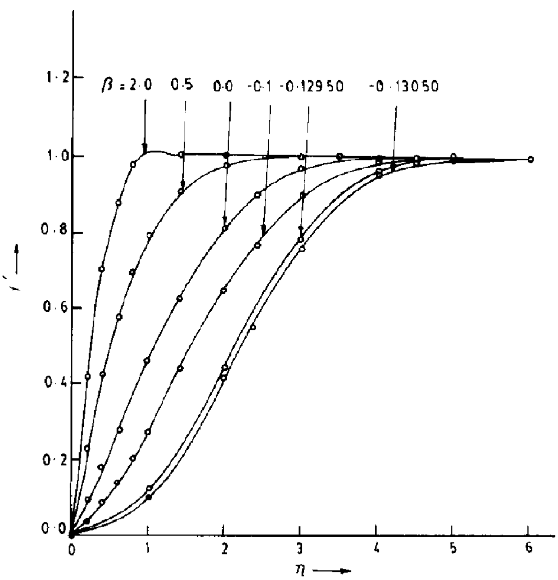

FIG. 6. Velocity distribution for a heated wall, $S_w = 1$.

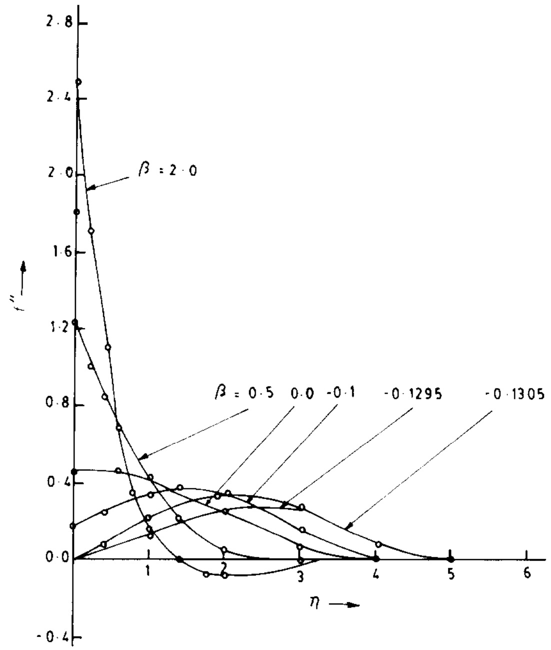

FIG. 7. Distribution of shearing stress for a heated wall, $S_w = 1$.

TABLE II

Variation of Heat-Transfer and Skin-Friction Parameters For Yawad Stagnation Line at Prandtl Number 0.7

$\frac{T_w}{T_0}$	$\frac{T_0}{T_{N0}}$	$f''(0)$	$g'(0)$	$\theta'(0)$	$\frac{f''(0)}{g'(0)}$
0.0	1.0	0.607487	0.497189	0.436376	1.22184
	1.6	0.687496	0.513058	0.422751	1.34000
	3.0	0.862536	0.544667	0.426361	1.58360
	6.5	1.254509	0.604459	0.458407	2.07542
0.5	1.0	0.936668	0.538367	0.469754	1.73983
	1.6	1.183120	0.569400	0.438761	2.07783
	3.0	1.702323	0.625490	0.435125	2.72158
	6.5	2.819874	0.720435	0.471887	3.91413

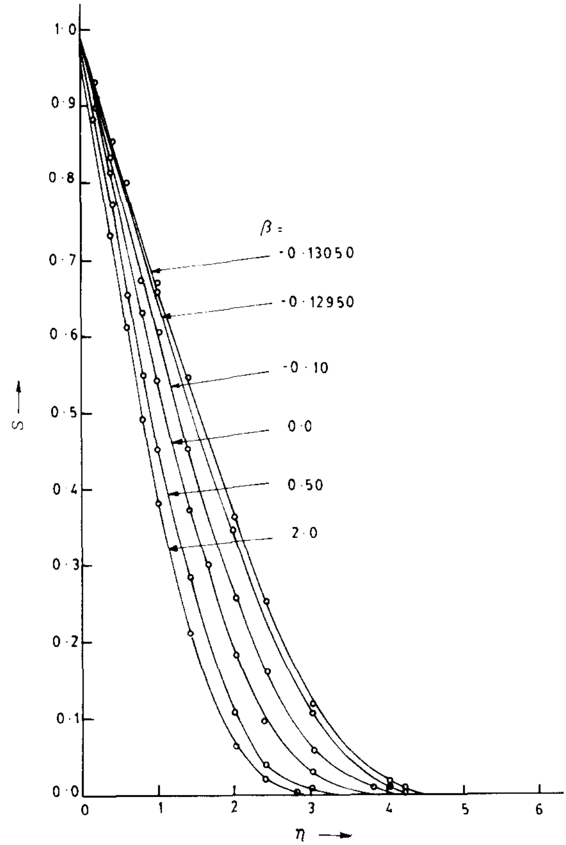

FIG. 8. Enthalpy distribution for a heated wall, $S_w = 1$.

CONCLUSION

It is to be remarked that the computational exercise pertaining to all the flow cases considered in this study, the iterative process, was always initiated with very crude and simple to obtain initial profiles given by (20) and the "continuation" as suggested by Robert and Shipman [18] was used to build the solutions for the subsequent sets of parametric values. All the results are obtained in at most seven iterations of the system (12) and in all cases an accuracy, as defined by Lee [12], of at least 10^{-10} is attained. On comparing the results of this study with those of Reshotko and Beckwith [17] one may conclude that the method of quasilinearization, when applicable, turns out to be quite a powerful, self-starting, easy to handle, and, moreover, reliable technique. As such, the numerical solutions of a non-linear two-point boundary-value problem of any order, obtained through the carryover of this very technique, may be accepted, without any reservation, whatsoever.

APPENDIX: NOTATION

A	Square matrix $[a_{ij}]$ of order 7
a	Sonic velocity
B	Column vector $[b_i]$ of order 7
C	Stagnation line chordwise velocity gradient
$\bar{c}$	Column vector of order 3
f	Function related to stream function ψ
g	Spanwise velocity variable
$\bar{H}$	Square matrix of order 3
H_3, H_5, H_7	Seven-dimensional column vectors
M	Mach number
m	Exponent from $U_e = CX^m$
P	Seven-dimensional column vector
$\bar{P}$	Column vector of order 3
Pr	Prandtl number
S	Enthalpy distribution
T	Static temperature
U	Transformed chordwise velocity component
u	Chordwise velocity component
X	Transformed chordwise coordinate
x	Chordwise coordinate
$X^T = (x_1, x_2, x_3, x_4, x_5, x_6, x_7)$	Seven-dimensional row vector
Z	Transformed normal coordinate
β	Pressure gradient parameter
γ	Ratio of specific heats
η	Similarity variable
θ	Normalized enthalpy function
Λ	Yaw angle
ν	Kinematic viscosity
ψ	Stream function

Subscripts

e	Local or external flow outside boundary layer
N	Component normal to cylinder axis
w	Wall or surface value
0	Free-stream stagnation value

Superscripts

n	Iteration number

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